THE INADEQUACY OF INPUT-OUTPUT INFORMATION IN ROBUST COMPENSATOR DESIGN

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Recent work by Vidyasagar [1], Khalil [2], Rohrs [3], and Ioannou and Kokotovic [4] has addressed the problems caused by unmodelled dynamics or parasitics in a closed-loop control system. We are specifically interested in the results of [1] where it is shown that, for any plant (A,B,C,D), any singularly perturbed augmentation of that plant

$$\begin{bmatrix} I & 0 \\ 0 & \varepsilon I \end{bmatrix}\dot{x} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}x + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}u$$

$$y = [\; C_1 \quad C_2 \;]x \tag{1a}$$

where

$$A_{11} - A_{12}A_{22}^{-1}A_{21} = A, \quad B_1 - A_{12}A_{22}^{-1}B_2 = B$$

$$C_1 - C_2A_{22}^{-1}A_{21} = C, \quad - C_2A_{22}^{-1}B_2 = D \tag{1b}$$

and any fixed strictly proper compensator, if the corresponding closed-loop system is stable at $\varepsilon = 0$, then so is the perturbed closed-loop system for sufficiently small $\varepsilon > 0$. We wish to explore the consequences of perturbing the compensator as well. Let (F,G,H,0) be any compensator and consider perturbations of the form (1).

<u>Theorem 1</u> Let $R < \infty$ and $\gamma > 0$. There exist $\varepsilon_0 > 0$ and singular perturbations (1) of the plant and compensator such that the closed-loop configuration of the perturbed systems has a pole p_ε satisfying $|p_\varepsilon| > R$ and $|\arg p_\varepsilon| < \gamma$ whenever $0 < \varepsilon < \varepsilon_0$.

Theorem 1 says that perturbations of plant and compensator can always be found which destabilize the closed-loop system. The proof of the theorem is based on examination of the perturbed closed-loop system

$$\begin{bmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & \varepsilon I & 0 \\ 0 & 0 & 0 & \varepsilon I \end{bmatrix}\begin{bmatrix} \dot{x} \\ \dot{z} \\ \dot{\xi} \\ \dot{\varsigma} \end{bmatrix} = \begin{bmatrix} A_{11} & B_1H_1 & A_{12} & B_1H_2 \\ G_1C_1 & F_{11} & G_1C_2 & F_{12} \\ A_{21} & B_2H_1 & A_{22} & B_2H_2 \\ G_2C_1 & F_{21} & G_2C_2 & F_{22} \end{bmatrix}\begin{bmatrix} x \\ z \\ \xi \\ \varsigma \end{bmatrix} + \begin{bmatrix} B_1 \\ 0 \\ B_2 \\ 0 \end{bmatrix}v$$

$$y = \begin{bmatrix} C_1 & 0 & C_2 & 0 \end{bmatrix} \begin{bmatrix} x \\ z \\ \xi \\ \varsigma \end{bmatrix}$$

It can be shown that A_{22}, B_2, C_2, F_{22}, G_2, and H_2 can always be found to make the matrix

$$\begin{bmatrix} A_{22} & B_2H_2 \\ G_2C_2 & F_{22} \end{bmatrix}$$

unstable; hence, the closed-loop system is unstable for $\varepsilon > 0$.

An important property of the perturbations (1) is that, in a certain sense, they exhibit the same input-output behavior as the nominal system. Specifically, let $\mathcal{U}$ = all C^1 functions $u:[0,\tau] \rightarrow \mathbb{R}^m$ with $u(0) = 0$, $\max \|u(t)\| < K_0$, and $\max \|\dot{u}(t)\| < K_1$ for some finite constants τ, K_1, and K_2, independent of u.

Theorem 2 If $\delta > 0$, there exists $\varepsilon_0 > 0$ such that, whenever $u \in \mathcal{U}$ and $0 \le \varepsilon < \varepsilon_0$, the respective outputs y and y_ε of (1) and (2) satisfy

$$\max_{0 \le t \le \tau} \|y(t) - y_\varepsilon(t)\| < \delta.$$

Interpreting the number δ as the minimum achievable measurement error, Theorem 2 shows that the nominal model (A,B,C,D) and the perturbed model (1) are equivalent in an input-output sense for sufficiently small ε. An analogous statement applies to the compensator. Hence, Theorems 1 and 2 combine to show that, if only input-output information is available in the plant and compensator models, stable closed-loop behavior can never be guaranteed.

[1] M. Vidyasagar, "Robust Stabilization of Singularly Perturbed Systems," *Systems and Control Letters*, 5 (1985), 413-418.

[2] H. K. Khalil, "A Further Note on the Robustness of Feedback Control Methods to Modelling Errors," *IEEE Trans. Auto. Control*, 29 (1984), 403-418.

[3] C. E. Rohrs, L. Valvani, M. Athans, G. Stein, "Robustness of Continuous-Time Adaptive Control Algorithms in the Presence of Unmodelled Dynamics," *IEEE Trans. Auto. Control*, (30) 1985, 881-889.

[4] P. Ioannou, P. V. Kokotovic, "Singular Perturbations and Robust Redesign of Adaptive Control," *Proc. 21st IEEE Conf. on Decision and Control*," Dec. 1982, 24-29.