ROBUST COMPENSATOR DESIGN BASED ON INPUT-OUTPUT INFORMATION

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SUMMARY

We are particularly interested in the ways in which unmodelled high-frequency or parasitic effects can influence the performance of a control system, once a compensation scheme has been devised and implemented. Several interesting examples have recently appeared in the literature (see e.g. [1] and [2]), illustrating how parasitics can drive a closed-loop system to instability even when a conventional analysis of the best available model of the system predicts stability.

One important feature of [1] and [2] is that closed-loop instability occurs only when a non-strictly proper compensator is used. To see that these difficulties might also occur when a strictly proper compensator is applied, consider the following system:

$$\begin{bmatrix} 1 & & & \\ & \varepsilon & & \\ & & \varepsilon & \\ & & & \varepsilon^4 \end{bmatrix} \dot{x} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & -2\varepsilon \\ 0 & \varepsilon & 1 & -3\varepsilon^2 \end{bmatrix} x + \begin{bmatrix} 2 \\ 0 \\ 0 \\ 1 \end{bmatrix} u \qquad (1)$$

$$y = [1 \quad 0 \quad 0 \quad -2]x$$

The system has eigenvalues equal to 1 and $-1/\varepsilon^2$, and as $\varepsilon \to 0^+$ the solutions of (1) converge to those of the $\varepsilon=0$ system in the sense of distributions and uniformly on compact subintervals of $(0, \infty)$, for every $x(0) \in \mathbb{R}^4$ and u in a broad class of admissible controls. The system (1) at $\varepsilon=0$ can be stabilized by applying the strictly proper compensator governed by

$$\dot{z} = -3z - 2y \qquad (2)$$

$$u = z$$

However, for $\varepsilon > 0$ the characteristic polynomial of the closed-loop system is non-Hurwitz and in fact can be shown to have at least one root with real part tending to $+\infty$ as $\varepsilon \to 0^+$. Furthermore, it can also be proven that the solutions of the composite system determined by (1) and (2) diverge for certain inital conditions. It should be noted that the nature of the example (1) is somewhat different from those considered in [1] and [2] in that (1) contains a multi-rate perturbation.

Although the situation illustrated by (1) and (2) is worse than previously thought, we are convinced that there exist ways of designing around such problems [3]. Part of the reason that robust designs are still possible is that models such as (1) with $\varepsilon=0$ contain information concerning the internal system structure. A natural question to ask is to what extend dropping all but input-output information affects our ability to do robust compensation.

To gain some insight, one might consider a system nominally described by

$$P(s) = \frac{2}{s-1} \tag{3}$$

and a compensator

$$C(s) = \frac{\alpha}{s+\beta} \tag{4}$$

One possible perturbation of P is

$$P_\varepsilon(s) = \frac{2}{s-1} + \frac{4\varepsilon^2}{\alpha} \cdot \frac{s^2}{(\varepsilon^2 s+1)^3}$$

This perturbation is physically reasonable since it is realized by

$$\begin{bmatrix} 1 & & & \\ & \varepsilon & & \\ & & \varepsilon & \\ & & & \varepsilon^4 \end{bmatrix} \dot{x} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & -2\varepsilon \\ 0 & \varepsilon & 1 & -3\varepsilon^2 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} u \tag{5}$$

$$y = [2 \quad 0 \quad 0 \quad \frac{4}{\alpha}]x$$

The system (5) is a slight generalization of (1), having the same eigenvalues and convergence properties. Again, it is easy to show that the characteristic polynomial of the closed-loop configuration is *non-Hurwitz* and has at least one root with real part tending to $+\infty$. Further, the corresponding solutions diverge for some initial conditions.

The important thing to noitce here is that a destabilizing perturbation of the form (5) exists for <u>any</u> first-order, strictly proper compensator (4). Hence, robust compensation cannot be achieved. We can prove a more general result:

<u>Theorem</u>. If the rational functions P(s) and C(s) are strictly proper and proper, respectively, there exists a parameterization $\{P_\varepsilon \mid \varepsilon \geq 0\}$ such that

1) the coefficients of P_ε converge to those of P as $\varepsilon \to 0^+$,

2) there is a singularly perturbed state-space realization of P_ε, each of whose eigenvalues is either fixed or equal to $-1/\varepsilon^p$ for some positive integer p and whose solutions converge in the sense of distributions and uniformly on compact subintervals of $(0, \infty)$ as $\varepsilon \to 0^+$ for every input and initial condition,

3) The closed-loop transfer function

$$H_\varepsilon = \frac{P_\varepsilon}{1-P_\varepsilon C}$$

has at least one pole λ_ε with $\text{Re}\,\lambda_\varepsilon \to +\infty$,

4) the sequence $y_\varepsilon = Cx_\varepsilon$ does not converge in the distribution sense as $\varepsilon \to 0^+$.

If the class of perturbations described in 1) - 4) is taken to be physically realizable, the theorem states that robust compensation can never be achieved if only input-output information is available. More research is needed to demonstrate the realizability of this class of perturbations.

REFERENCES

[1] H. K. Khalil, "On the Robustness of Output Feedback Control Methods to Modelling Errors," IEEE Trans. Auto. Control, 26, April 1981.

[2] H. K. Khalil, "A Further Note on the Robustness of Output Feedback Control Methods to Modelling Errors," IEEE Trans. Auto. Control, 29, September 1984.

[3] D. Cobb, "Slow and Fast Stability in Singular Systems," 23rd IEEE Conf. on Decision and Control, 1984.