SMALL-SIGNAL THIRD-ORDER DISTORTION ANALYSIS OF TRANSISTOR AMPLIFIERS†

J. D. COBB AND T. N. TRICK
University of Illinois
Urbana, Illinois 61801

I. INTRODUCTION

Recently there has been considerable interest in the effects of biasing on the distortion characteristics of transistor amplifiers [1-3]. Specifically, a virtual elimination of third order distortion has been observed in some bipolar transistor configurations by selection of the operating point.

In this paper two different mechanisms which can bring about cancellation of third order distortion are isolated and shown to be intimately related. They are the base-emitter junction diode and the avalanche nonlinearities.

II. BASE-EMITTER NONLINEARITY

Consider the simple low-frequency model of an npn transistor as shown in figure 1. It is assumed that the transistor is operating in the active region at all times. This model contains only the base-emitter nonlinearity or exponential characteristic described by

$$v_{BE} = V_T \ln \left(\frac{\beta i_B}{I_S} + 1\right) \simeq V_T \ln \frac{\beta i_B}{I_S} \tag{1}$$

which can readily be expanded in a Taylor series about the operating point,

$$v_{BE} = \sum_{n=o}^{\infty} b_n (i_B - I_B)^n \tag{2}$$

$$b_n = \begin{cases} V_T \ln \dfrac{\beta I_B}{I_S}, & n = o \\ (-1)^{n+1} \dfrac{V_T}{n I_B^n}, & n > o \end{cases} \tag{3}$$

Capital letters with upper case subscripts denote operating point values.

If we define the small signal perturbations

$$v_{be} \triangleq v_{BE} - V_{BE} \tag{4}$$

†This work was supported by the National Science Foundation under Grant ENG 75-02708.

$$i_b \triangleq i_B - I_B \tag{5}$$

then we can write

$$v_{be} = \sum_{n=1}^{\infty} b_n i_b^n \quad . \tag{6}$$

This corresponds to shifting the origin to the operating point.

Let us represent the nonlinear terms by

$$v_e \triangleq \sum_{n=2}^{\infty} b_n i_b^n \tag{7}$$

then this suggests the small signal model shown in figure 2. Note that the only nonlinear element is the distortion generator v_e.

The test circuit in figure 3 was used to study distortion as a function of the operating point due to the base-emitter junction. As a measure of third order distortion the quantity $|\frac{H_3}{H_1}|$ was calculated where H_3 and H_1 are the third and first order transfer functions respectively from the Volterra series method of nonlinear analysis [4].

In order to calculate $|\frac{H_3}{H_1}|$ for the specific circuit in question, the exponential input method was used. Briefly, the derivation goes as follows.

Let the input

$$v_s(t) = e^{S_1 t} + e^{S_2 t} + e^{S_3 t} \quad . \tag{8}$$

Then the coefficient of the $e^{(S_1+S_2+S_3)t}$ term at the output is H_3. To find this coefficient, first short the distortion generation v_e. It follows that

$$i_b(t) = \frac{1}{R_S+b_1}\,(e^{S_1 t} + e^{S_2 t} + e^{S_3 t}) \quad . \tag{9}$$

The coefficient of each of the second order modes of v_e is then

$$\frac{2b_2}{(R_S+b_1)^2} = -\frac{V_T}{I_B^2(R_S+\frac{V_T}{I_B})^2} \quad . \tag{10}$$

The coefficient of the $e^{(S_1+S_2+S_3)t}$ mode of v_e is

$$\frac{6b_3}{(R_S+b_1)^3} = \frac{2V_T}{I_B^3(R_S+\frac{V_T}{I_B})^3} \quad . \tag{11}$$

Next, find the coefficient of the third order mode v_e due to the interaction of second order and first order terms. The coefficient of each of the second order terms of i_b is

$$-\frac{2b_2}{(R_S+b_1)^3} = \frac{V_T}{I_B^2(R_S+\frac{V_T}{I_B})^3} \quad . \tag{12}$$

The third order mode of v_e due to this interaction then has the coefficient

$$6b_2\left[-\frac{2b_2}{(R_S+b_1)^3}\right](\frac{1}{R_S+b_1}) = -\frac{3V_T^2}{I_B^4(R_S+\frac{V_T}{I_B})^4} \quad . \tag{13}$$

Finally, short the input generator v_S and find the coefficient of the third order term at the output.

$$H_3 = \frac{\beta I_B V_T R_L(2I_B R_S - V_T)}{(I_B R_S + V_T)^5} \tag{14}$$

By inspection

$$H_1 = -\frac{\beta I_B R_L}{(I_B R_S + V_T)} \quad . \tag{15}$$

Therefore

$$\left|\frac{H_3}{H_1}\right| = \frac{V_T|2I_B R_S - V_T|}{(I_B R_S + V_T)^4} \quad . \tag{16}$$

If

$$v_s(t) = A \cos wt \tag{17}$$

then there are contributions to the cos 3 wt mode at the output due to all third and higher odd order transfer functions. If A is small enough then higher order contributions can be neglected and the third order component of the output voltage is

$$\frac{1}{2^2}(\frac{1}{3!}) A^3 H_3 = \frac{A^3}{24} H_3 \quad . \tag{18}$$

If we define $v_o^{(3)}$ and $v_o^{(1)}$ as the third harmonic and fundamental rms amplitudes of the output voltage, then

$$\frac{v_o^{(3)}}{v_o^{(1)}} = \left|\frac{\frac{A^3}{24} H_3}{AH_1}\right| = \frac{A^2}{24}\left|\frac{H_3}{H_1}\right| . \tag{19}$$

Clearly, for constant input level, $\left|\frac{H_3}{H_1}\right|$ is approximately proportional to $\frac{v_o^{(3)}}{v_o^{(1)}}$ and is therefore a valid measure of normalized third order distortion.

If we set

$$I_B = \frac{V_T}{2R_S} \tag{20}$$

then

$$\left|\frac{H_3}{H_1}\right| = 0 \tag{21}$$

and third order distortion is virtually eliminated. The ratio $\left|\frac{H_3}{H_1}\right|$ is plotted in figure 4.

Measurements were made in the laboratory which seem to verify these results. Data was taken for several values of load resistance. All readings were taken with A = 28.3 mV. Figure 5 shows the relationship between measured and theoretical values. Note the characteristic drop in distortion near the collector current corresponding to a base current of $\frac{V_T}{2R_S}$.

In figure 6 a similar set of curves is plotted, this time for a larger source resistance. Again a striking reduction in distortion can be observed near the predicted value of collector current.

The slight inconsistencies between predicted and measured values are probably due to problems in the experimental set up. Predicted values were calculated assuming an operating temperature of 25°C while the actual temperature of operation may have been somewhat higher, accounting in part for the obvious shift in the position of the point of minimum distortion. Also, some distortion was introduced by the source generator.

It should be noted that the actual levels of distortion are very close to those predicted. This indicates the well-known dominance of the base-emitter nonlinearity in the active region. However, the obvious conclusion, as far as third order distortion is concerned, is that this dominance can be eliminated by correctly biasing the base-emitter junction, leaving open the possibility of significant interactions between two or more nonlinear effects. These ideas serve as motivation for the next section.

III. AVALANCHE AND BASE-EMITTER INTERACTIONS

Consider the transistor model shown in figure 7. Again the base-emitter nonlinearity is included. In this model, however, the dependent current source takes into account the avalanche breakdown effect of the collector-base junction which is governed by the equation

$$i_C = \frac{\beta i_B}{1 - \left(\frac{V_{CB}}{V_{CBO}}\right)^4} \quad . \qquad (22)$$

V_{CBO} is the collector-base breakdown voltage of the transistor.

This expression can be expanded in a Taylor series about the operating point

$$i_C = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} a_{nm} (i_B - I_B)^n \, (v_{CB} - V_{CB})^m \quad . \qquad (23)$$

It is convenient that

$$a_{nm} = 0 \; , \quad n \geq 2 \quad . \qquad (24)$$

If we define

$$v_{cb} \triangleq v_{CB} - V_{CB} \qquad (25)$$

$$i_c \triangleq i_C - I_C \qquad (26)$$

then

$$i_c = \sum_{m=1}^{\infty} a_{om} v_{cb}^m + \sum_{m=0}^{\infty} a_{1m} i_b v_{cb}^m \quad . \qquad (27)$$

Letting

$$i_e \triangleq \sum_{m=2}^{\infty} a_{om} v_{cb}^m + \sum_{m=1}^{\infty} a_{1m} i_b v_{cb}^m \qquad (28)$$

suggests the small signal model of figure 8.

Becuase of the complexity of even a simple model such as this, a digital computer was used to perform the analysis. Figure 9 shows a typical pair of curves generated by adjusting collector-base voltage while holding base current constant. Null points can be observed for various values of bias voltage. The breakdown voltage of the transistor is approximately 40 v.

In general there seems to be two ranges of base current at which these nulls occur. The position of the null is plotted versus base current in figure 10.

At first it may seem rather strange that there can be a complete cancellation between base-emitter distortion and the normally less significant avalanche nonlinearity. However, the reason for this sort of interaction can be readily seen if one observes that the regions where nulls exist correspond exactly to those regions where distortion due to the base-emitter nonlinearity alone is small. Thus, if the transistor is biased with an appropriate base current, the third order distortion due to the base-emitter nonlinearity alone is small enough for a null to exist at some collector-base voltage. The complete cancellation is due to interaction of the two nonlinear effects.

It is interesting to note that the slope of the curve in figure 10 is always negative. This indicates that the collector-base voltage at which the null occurs can be decreased by increasing the base current.

These results seem to be true in general. The position of the null can be placed almost anywhere by adjusting the base current. The two regions in which nulls occur can be positioned by adjusting the various parameters in the circuit. They can even be made to overlap giving two nulls at the same base current.

IV. CONCLUSION

It has been shown that third order distortion can be reduced by selecting an appropriate operating point. First, the base-emitter junction should be biased to bring down the level of distortion due to the base-emitter nonlinearity alone. Then, the collector-base voltage can be adjusted to bring about a further reduction in distortion due to the interaction of the two nonlinear effects.

V. REFERENCES

[1] J. Eachus, "Distortion in Ultralinear Solid-State Devices," IEEE Journal of Solid-State Circuits, Vol. SC-10, pp. 485-497, Dec. 1975.

[2] L. C. Thomas, "Eliminating Broadband Distortion in Transistor Amplifiers," Bell System Technical Journal, March 1968.

[3] A. J. Mallinckrodt, F. M. Gardner, "Distortion in Transistor Amplifiers," IEEE Transactions on Electron Devices, pp. 288-289, July 1963.

[4] S. Narayanan, "Transistor Distortion Analysis Using Volterra Series Representation," Bell System Technical Journal, Vol. 46, pp. 991-1023, May-June 1967.

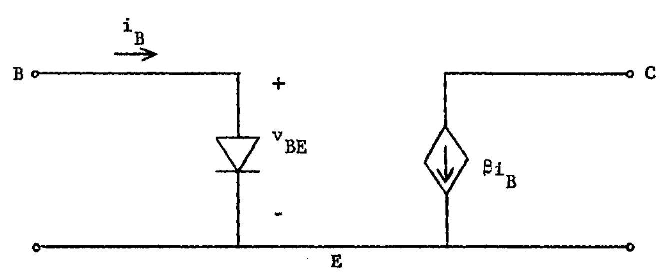

Figure 1. Low frequency model with base-emitter nonlinearity only.

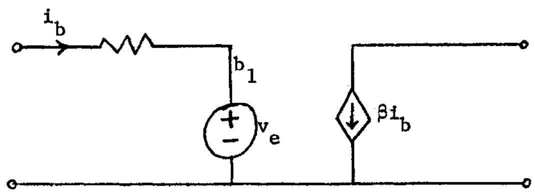

Figure 2. Small signal model.

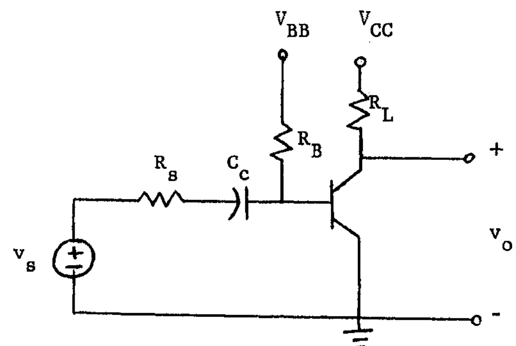

Figure 3. Test circuit.

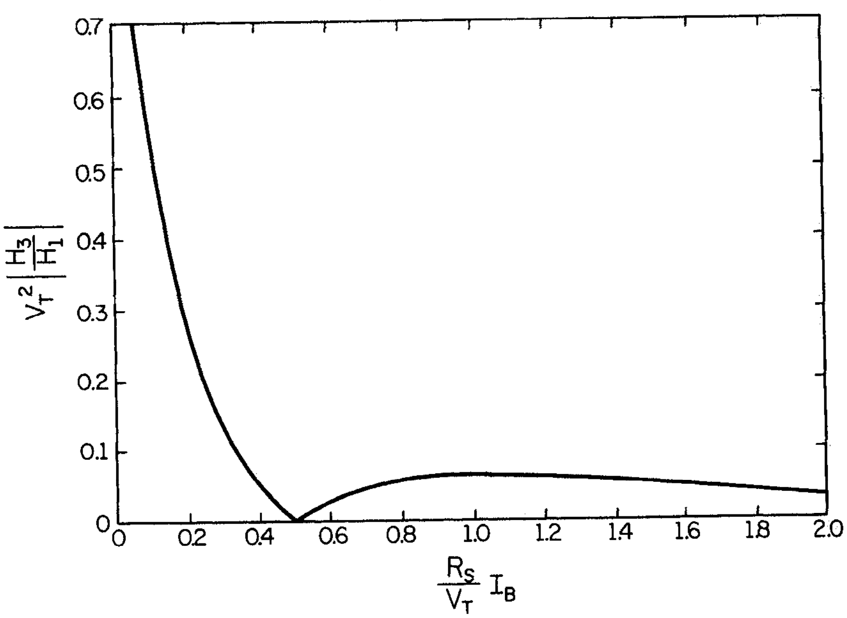

Figure 4. Third order distortion versus base current.

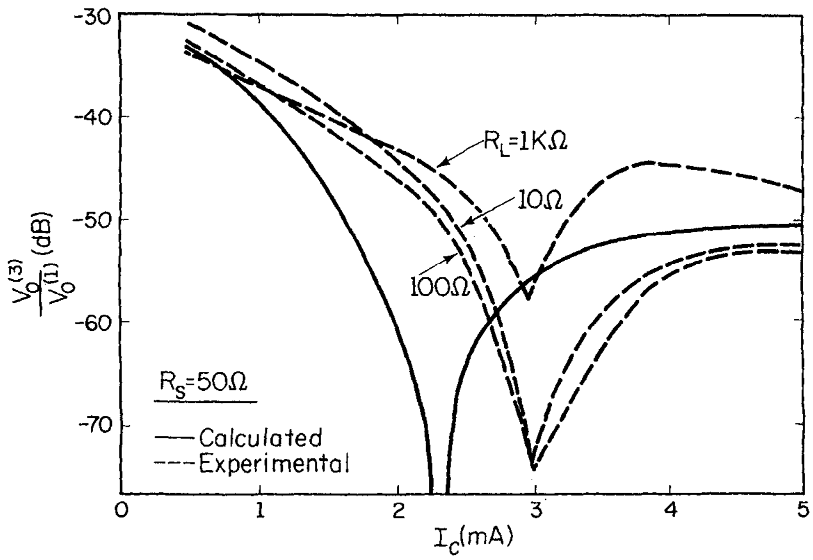

Figure 5. Measured and theoretical distortion for R_s = 50Ω .

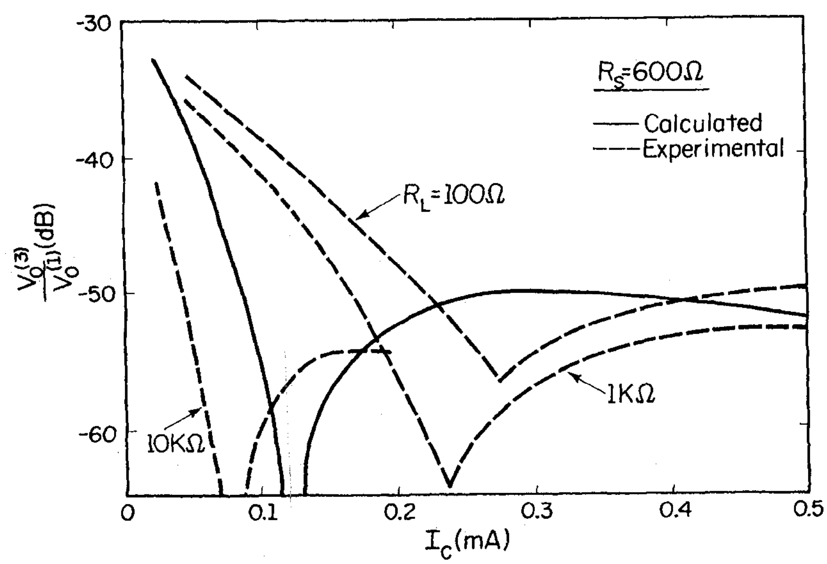

Figure 6. Measured and theoretical distortion for R_s = 600Ω.

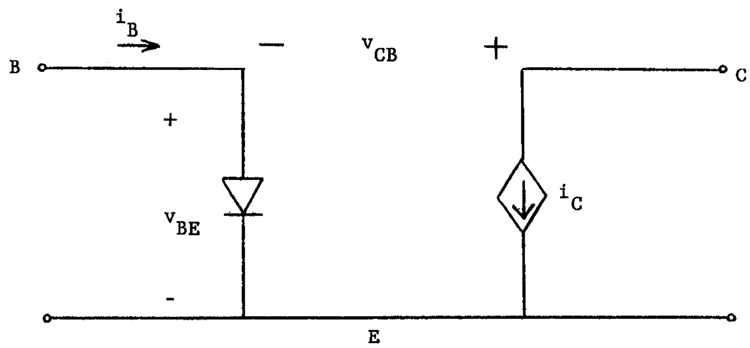

Figure 7. Low frequency model with avalanche effect included.

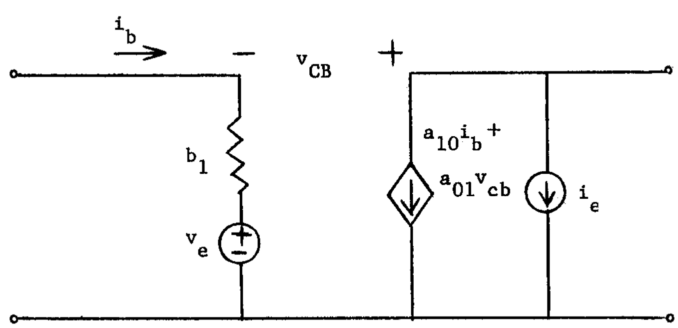

Figure 8. Small signal model.

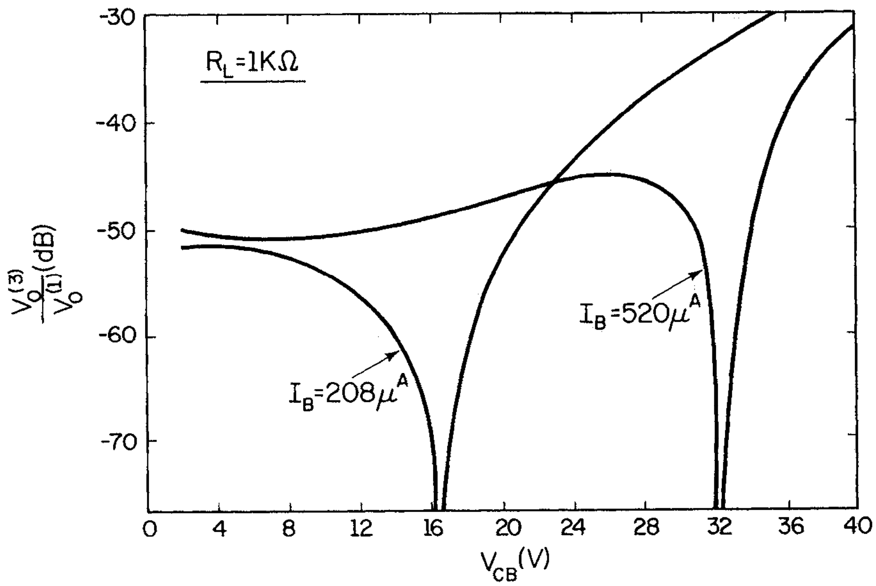

Figure 9. Third order distortion versus collector-base voltage.

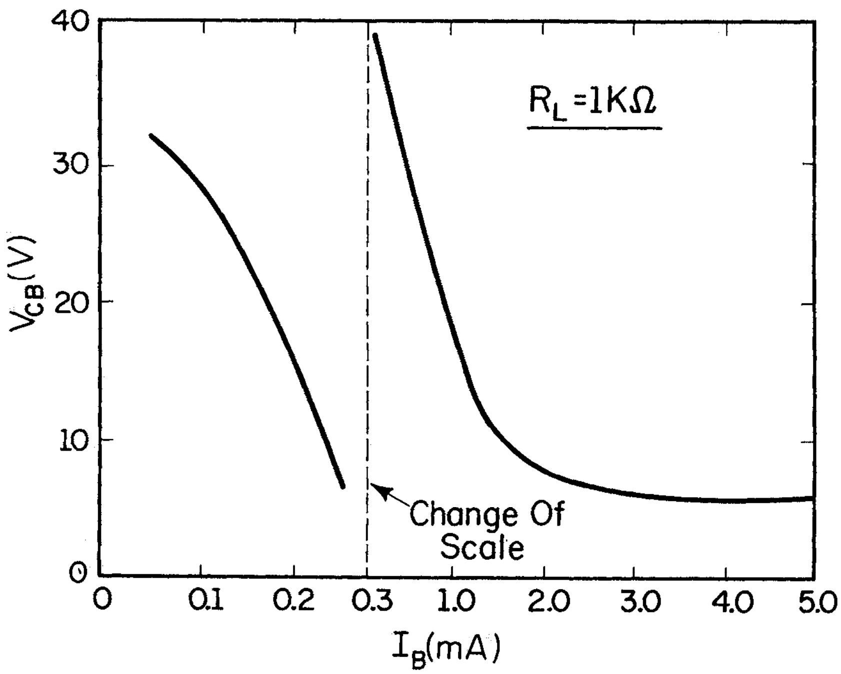

Figure 10. Position of null versus base current.